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MASS TRANSFER IN AGGREGATED POROUS MEDIA WITH NONLINEAR COMPRESSIBILITY

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The problem of the squeezing out of a solution from aggregates in a porous medium has been considered and investigated. A specific feature of the problem lies in taking account of the existence of two compaction zones in deforming aggregates of a porous medium. Features essential to mass transfer in a deformable aggregated porous medium have been determined and analyzed.

Introduction. Knowledge of the specific features of deformation and mass transfer in aggregated porous media is of great importance for studying mass transfer in both technology (the manufacture of medicines and constructional materials as well as the extraction) and natural systems (clay rocks, soils, and peats). In most cases, aggregates of a porous medium are of polydisperse structure: aggregates consist of fairly large particles whose interspace is filled to a variable degree with smaller particles. Such a structure is typical of some types of medical preparations, clays, and soils. Evidently, with deformation of such a medium between intra- and interaggregate pores, mass transfer proceeds in two stages. In the first stage, which is characterized by a higher permeability coefficient of the interaggregate space, a solution is squeezed out from the aggregate pores not completely occupied by finely dispersed particles. In the second stage, all intra-aggregate pores are filled with a finely dispersed material and the squeezing out from aggregates occurs due to the forcing out of a solution from the pores between finely dispersed particles. Thus, a need arises for solving the problem of nonuniform shrinkage of an aggregated porous medium where the existence of two compaction zones in an aggregate and a movable boundary between them should be taken into account. A similar problem has already been posed and solved previously [1, 2] in the study of rheological properties of clays. Here, consideration will be given to the squeezing out of a solution from spherical aggregates, which involves the formation of a zone of complete compaction in them. Below, a distinction will be made between the flow over a free part of the aggregate pores (not occupied by finely dispersed particles) and the flow through the compacted zone. Evidently, both zones differ primarily in the permeability coefficient, since in the second case the flow resistance resulting from the passage, through the system, of finely dispersed particles in the compacted zone is much higher than in the first.

In both cases, filtration is described using the equation of an elastic regime [3]. According to the concepts developed in [1, 2], two zones of the solution shrinkage and squeezing out from aggregates are formed, namely, a "fast" and a "slow" one from the viewpoint of the filtration velocity in them, with different coefficients χ_1 and χ_2 (Fig. 1). When shrinkage in a "fast" zone reaches a certain value [1, 2], the coefficient χ_1 of a "fast" zone assumes stepwise the value of χ_2 of a "slow" zone; therefore, the boundary between the two zones is movable (Fig. 2). The arising problem for the shrinkage and pressure in an aggregate is the problem formulated by N. N. Verigin [4].

Formulation of the Problem. The equation for filtration in the interaggregate space of a porous layer is of the form

$$\frac{\partial p}{\partial t} = \chi_{\text{b}} \frac{\partial^2 p}{\partial x^2} + j\,, \quad j = \sigma \chi_2 \left.\frac{\partial w_2}{\partial R}\right|_{R=a} . \tag{1}$$

Pressure is specified on the upper and lower boundaries of the layer. The type of the source in Eq. (1) was sought considering N. N. Verigin's problem of filtration in a spherical aggregate (Fig. 2) incorporating two compaction zones with constant but dissimilar values of the piezoconductivity and with a moving boundary ξ between the zones (the shrinkage value is prescribed on the latter)

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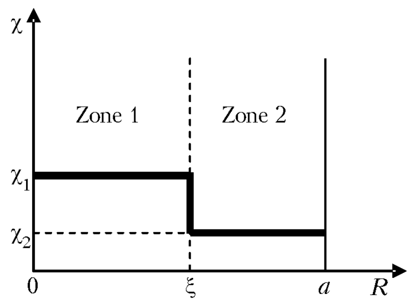

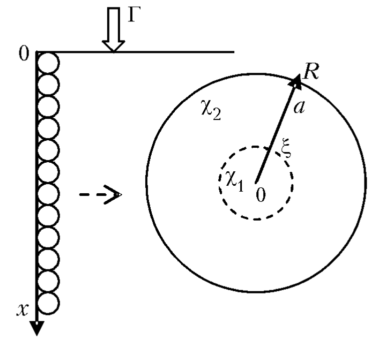

Fig. 1. Formation of two shrinkage zones in an aggregate with various values of the piezoconductivity and a movable boundary between them.

Fig. 2. Structure of a layer of a porous medium containing aggregates.

$$\frac{\partial w_1}{\partial t} = \chi_1 \frac{\partial \left(R^2 \frac{\partial w_1}{\partial R} \right)}{\partial R}, \quad 0 \le R \le \xi ; \tag{2}$$

$$\frac{\partial w_2}{\partial t} = \chi_2 \frac{\partial \left(R^2 \frac{\partial w_2}{\partial R} \right)}{\partial R}, \quad \xi \le R \le a . \tag{3}$$

We now write the boundary conditions for an aggregate:

$$\left. \frac{\partial w_1}{\partial R} \right|_{R=0} = 0 ; \quad w_2 (R, x, t) = p (x, t) . \tag{4}$$

The initial pressure distribution and the condition on the boundary of the two zones are assumed as

$$w_1 (R, x, 0) = \Gamma ; \quad \chi_1 \left. \frac{\partial w_1}{\partial R} \right|_{R=\xi} = \chi_2 \left. \frac{\partial w_2}{\partial R} \right|_{R=\xi}, \quad w_1 (\xi, x, t) = w_2 (\xi, x, t) = \text{const} . \tag{5}$$

The equation for concentration transfer in the layer is of the form [4]

$$m \frac{\partial C}{\partial t} + \frac{\partial}{\partial x} (CV) = \beta c_0 j , \quad j = \sigma \chi_2 \left. \frac{\partial w_2}{\partial R} \right|_{R=a}, \quad V = - \chi_b \beta \frac{\partial p}{\partial x} . \tag{6}$$

Presumably, there is no concentration source on the upper boundary of a porous layer, and at the initial time instant the concentration in interaggregate pores is zero.

We now introduce the notation $T\tau = t$, $\zeta a = \xi$, $ra = R$, $U\Gamma = w$, $\Pi\Gamma = p$, and $yH = x$. The choice of T is significant. Let $T = a^2/\chi_b$; hence, a situation of "short times" is modeled, since χ_b has the largest value ($\chi_b/\chi_1 >> 1$, $\chi_b/\chi_2 >> 1$). The above substitutions in Eqs. (1)–(6) and simple rearrangements give the equation of filtration in the layer

$$\frac{\partial \Pi}{\partial \tau} = F \frac{\partial^2 \Pi}{\partial y^2} + E \frac{\alpha - \Pi}{\bar{\zeta}} \tag{7}$$

with boundary and initial conditions

$$\Pi(0,\tau)=1\,,\quad \Pi(1,\tau)=0\,,\quad \Pi(y,0)=1\,. \tag{8}$$

Here, $E = a\sigma\chi_2/\chi_b$, $F = a^2/H^2$, and $\zeta = \zeta^{-1}$ (the meaning of introducing this parameter is considered below). For the problem in an aggregate we have

$$\frac{\partial U_1}{\partial \tau}=\frac{\chi_1}{\chi_b}\frac{1}{r^2}\frac{\partial\left(r^2\frac{\partial U_1}{\partial r}\right)}{\partial r}\,,\quad 0\le r\le\zeta\,;\quad \frac{\partial U_2}{\partial \tau}=\frac{\chi_2}{\chi_b}\frac{1}{r^2}\frac{\partial\left(r^2\frac{\partial U_2}{\partial r}\right)}{\partial r}\,,\quad \zeta\le r\le 1\,.$$

Evidently, $\frac{\partial U_1}{\partial \tau}$, $\frac{\partial U_2}{\partial \tau}\to 0$. Substituting $1/r = z$, for an aggregate we obtain

$$\frac{\partial^2 U_1}{\partial z^2}=0\,,\quad \bar{\zeta}\le z\le\infty\,,\quad \frac{\partial^2 U_2}{\partial z^2}=0\,,\quad 1\le z\le\bar{\zeta}\,.$$

Therefore, from the relation $\chi_1\frac{\partial U_1}{\partial z}\bigg|_{z=1/\zeta} = \chi_2\frac{\partial U_2}{\partial z}\bigg|_{z=1/\zeta}$ we obtain

$$U(\bar{\zeta},\tau)=\text{const}=\alpha\,.$$

We now present the condition for the boundary after averaging over z:

$$\frac{d\bar{\zeta}}{d\tau}=\gamma\frac{\alpha-\Pi}{\bar{\zeta}}\,,\quad \gamma=\frac{\chi_2\beta\Gamma}{\chi_b}\,, \tag{9}$$

whence

$$\frac{\partial U_2}{\partial R}\bigg|_{R=1}=\frac{\alpha-\Pi}{\bar{\zeta}}\,,\quad \bar{\zeta}=\sqrt{2(\alpha-\Pi)\gamma\tau}\,.$$

Then, the boundary and initial conditions take the form

$$\bar{\zeta}(0,\tau)=\sqrt{2(\alpha-1)\gamma\tau}\,,\quad \bar{\zeta}(1,\tau)=\sqrt{2\alpha\gamma\tau}\,,\quad \bar{\zeta}(y,0)=\sqrt{2(\alpha-1)\gamma\tau}\,. \tag{10}$$

Combining Eqs. (9) and (10) gives a single equation

$$\frac{\partial\bar{\zeta}^2}{\partial\tau}=F\frac{\partial^2\bar{\zeta}^2}{\partial y^2}-2E\bar{\zeta}+A\,,\quad A=\text{const} \tag{11}$$

with boundary and initial conditions

$$\bar{\zeta}(0,\tau)=\sqrt{2(\alpha-1)\gamma\tau}\,,\quad \bar{\zeta}(1,\tau)=\sqrt{2\alpha\gamma\tau}\,,\quad \bar{\zeta}(y,0)=\sqrt{2(\alpha-1)\gamma\tau}\,. \tag{12}$$

For the concentration in the layer we have

$$m\frac{\partial c}{\partial\tau}-FG\frac{\partial}{\partial y}\left(c\frac{\partial\Pi}{\partial y}\right)=EG\frac{\alpha-\Pi}{\bar{\zeta}}\,, \tag{13}$$

where $G = \beta\Gamma$, with the boundary and initial conditions

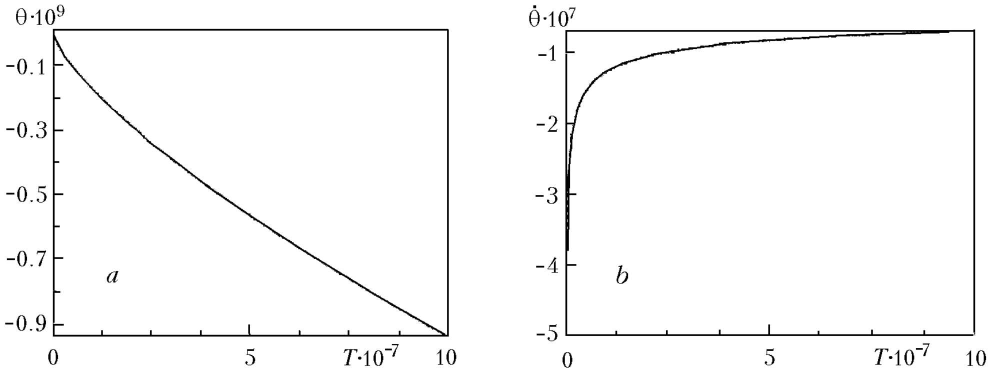

Fig. 3. Plots of the average shrinkage (a) and average-shrinkage rate (b) as functions of dimensionless time.

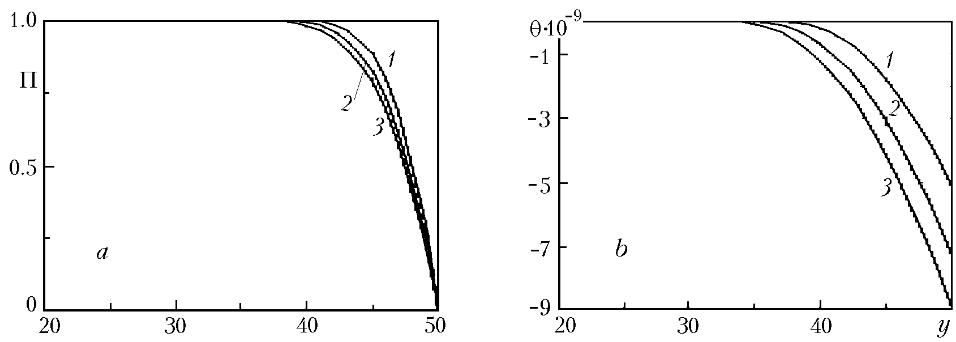

Fig. 4. Distribution of pressure (a) and shrinkage (b) in a layer: 1, 2, and 3) at $\tau = T/3$, $2T/3$, and T, respectively.

$$\left.\frac{\partial c}{\partial y}\right|_{y=0} = 0\,, \quad c\,(y, 0) = 0\,. \tag{14}$$

The model presented allows one to solve yet another important problem, namely, the problem of the layer shrinkage. The equation for shrinkage [3]

$$\frac{\partial \theta}{\partial t} + \mathrm{div}\,\mathbf{q} = 0$$

in the case considered takes the form

$$\frac{\partial \theta}{\partial t} = \frac{k}{\eta}\frac{\partial^2 p}{\partial x^2}\,.$$

In dimensionless variables, using simple rearrangements we get

$$\theta = T\frac{\chi\beta\Gamma}{H^2 F}\left(\Pi - \frac{E}{\gamma}\bar{\zeta} + \delta_0\right), \quad \delta_0 = \mathrm{const}\,. \tag{15}$$

Calculated Results. Figure 3 shows time dependences of the average shrinkage and average shrinkage rate. Figure 4 presents the pressure and shrinkage distributions in the layer for various time instants, and Fig. 5 shows the concentration distribution in the layer for various time instants and the curve of the time variation of the impurity concentration on the lower boundary of the layer. Calculations were performed for $T \le 10^8$. The shape of the curves in

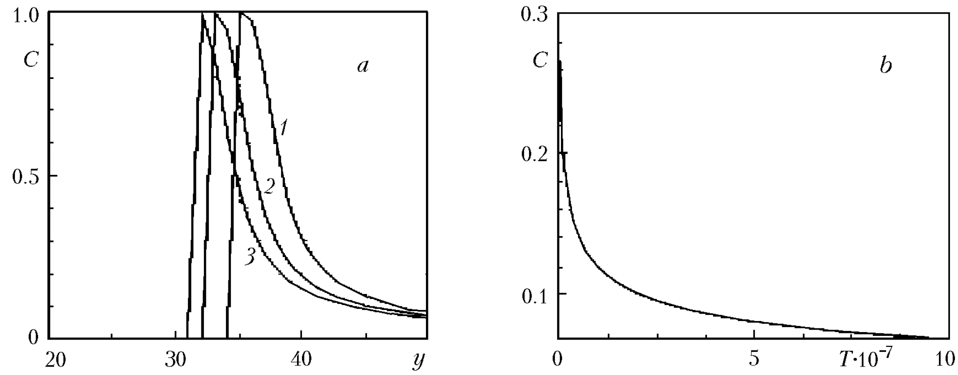

Fig. 5. Distribution of the impurity concentration in a layer (a, the number of aggregates in a layer is 50; 1, 2, and 3) at $\tau = T/3$, $2T/3$, and T, respectively) and of the impurity concentration at the outlet from a layer (b).

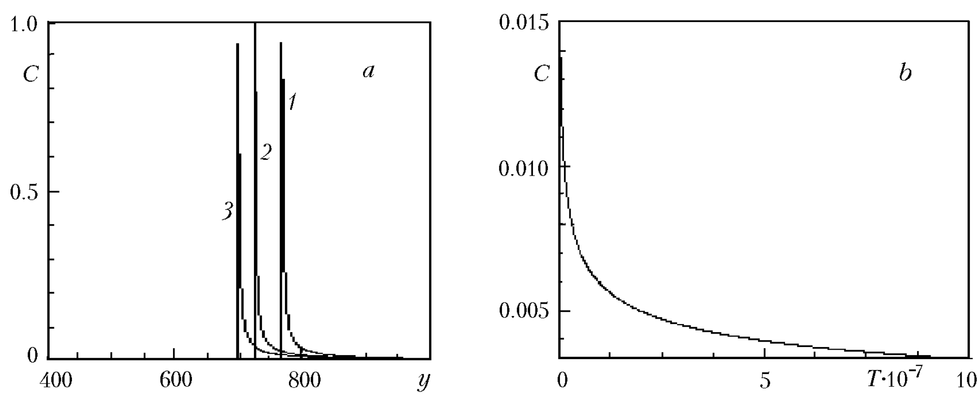

Fig. 6. Distribution of the impurity concentration in a layer (a, the number of aggregates in a layer is 1000; 1, 2, and 3) at $\tau = T/3$, $2T/3$, and T, respectively) and of the impurity concentration at the outlet from a layer (b).

Fig. 5 reguires additional comments. It appears that nonuniform aggregate shrinkages affect the dynamics of mass transfer in the layer. At each time instant, a solution is squeezed out from the aggregates, confirming a nonzero flow on the boundary of the aggregates. The process of squeezing out spreads upward. Here, a zone is formed with impurities in its interaggregate space still absent. Its boundary moves upward as a solution is squeezed out from aggregates into the interaggregate space. From Fig. 5 it is seen that a solution is squeezed out from the aggregates gradually.

We now turn to the question as to the influence of the selected values of parameters, primarily of the full size of an aggregate, on the form of the relations obtained. To this end, calculations were performed for values of parameters at which the layer is "more compacted" (the number of particles in the layer is 1000 rather than 50). The form of the time dependences of the average shrinkage and average shrinkage rate did not markedly change, nor did the shape of curves of the impurity concentration in the layer for various time instants (see Fig. 6a) and for the concentration at the outlet from the layer (see Fig. 6b). Since the aggregate radius became smaller, the shrinkage zone in the aggregates diminished; Therefore, the volume of a solution flowing from the layer decreased.

Conclusions. The study of the model of mass transfer in structured (aggregated) porous media indicated that the squeezing out of a solution from a porous layer involves a nonuniform shrinkage of a layer. Here, the squeezing out of an impurity decreases steadily, though over a fairly large time interval. The obtained profiles of the impurity concentration in the layer show that initially the process occurs in the lowest layers and thereafter proceeds ever higher up the layer to form a shrinkage zone of a porous layer. The trends determined allow the calculation of various mass-transfer regimes for various technological and natural processes.

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NOTATION

A, E, F, and G, constants; a, aggregate radius; c_0, solution concentration in an aggregate; C, solution concentration in a layer; c, normalized concentration in a layer; j, magnitudes of the inflow from an aggregate to a layer; H, thickness of a porous layer; k, permeability of clays; m, layer porosity; p, water pressure in a layer; $\mathbf{q}$, vector of filtration velocity; R, coordinate of a point in an aggregate; r, normalized intra-aggregate radius; T, total time of the rheological experiment; t, time; U, normalized pressure in an aggregate; V, filtration velocity in a layer; w_1, pressure in an incompletely compacted zone of an aggregate; w_2, pressure in a compacted zone of an aggregate; x, coordinate in a layer; y, normalized coordinate in a layer; z, reciprocal normalized radius; α, pressure on the boundary between the zones in an aggregate; β, compressibility coefficient; Γ, load; γ and δ_0, constants; ζ, normalized boundary of contact of two zones in an aggregate; η, viscosity of water; θ, shrinkage; $\dot{\theta}$, shrinkage rate; χ, piezoconductivity; χ_1, piezoconductivity in an incompletely compacted zone of an aggregate; χ_2, piezoconductivity in a completely compacted zone of an aggregate; χ_b, piezoconductivity of a layer; ξ, boundary of contact of two shrinkage zones; Π, normalized pressure in a layer; σ, specific surface of an aggregate; τ, normalized time of the process. Subscripts: 0, value of invariable parameters; 1, parameters of the first shrinkage zone in an aggregate; 2, parameters of the second shrinkage zone in an aggregate; b, interaggregate space; dot above a symbol, partial time derivative; overbar, reciprocal normalized boundary of contact of two zones in an aggregate.

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